Electrodynamical interactions inside a system of nano-particles

V. Lozovski*

Institute of Semiconductor Physics, National Academy of Sciences of Ukraine, pr. Nauki 45, Kyiv-28, 03028, Ukraine

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Abstract

A self-consistent scheme for calculation of the effective susceptibility of a many-body nano-system is proposed. The scheme is based on the idea of pseudo-vacuum Green function. The recurrent expressions connecting the Green function of the \( n-1 \) particle system with that of \( n \) particle system are obtained. The general sum rule for the effective susceptibilities of a many-body system is discussed.

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1. Introduction

The development of nano-technologies has led to the necessity of consideration of electrodynamics of small nonpointness particles. Especially this problem became actual for consideration of quantum dot systems [1–3], development of scanning near-field optical microscopy [4–6] and the electrodynamics of nano-composite systems [7,8]. In any aspect of these one can point to some achievements in solving these problems. For example, some ideas of the local field were used as a base for numerous works devoted to nonlocal electrodynamics of quantum well [9–11] and quantum dot [12–14] structures. The problem for single particles is solved in most of these works. In practice, however, we deal usually with the systems of nano-particles. In the case in which concentration of the particles is rather low one can neglect inter-particle interaction. However, in many cases this interaction cannot be neglected. For example, the development of scanning near-field optical microscopy requires often to take into account the electrodynamical interactions between an object and a probe. Moreover, these interactions can be resonant [15]. This means that electromagnetic interactions between the object and probe are very important. The interaction between the particles in nano-composite systems has to be taken into account especially for the dense ones. The most convenient method for dealing with this problem is the Green function method, as well as development of this method for electrodynamics of small particles, the local field method. The perfect review of these methods is given in the work [16]. One can note that the problem of taking into account the interactions inside the system of nano-particles has two aspects. First, the property of the particles being not point-like has to be taken into account. This allows us to know the spatial distribution of the local field inside each of the particle. Second aspect of the problem is taking into
account inter-particle interactions. These interactions could be taken into account by the use of generalized Green function. That is the Green function of the effective medium in which all particles in the system except the particle under consideration (let this particle be named as “nth” particle) are included. This method is based on the general idea of pseudo-vacuum Green function [16], which reduces the many-body problem to the problem of a single particle situated in some effective medium. The problem of connection between the external (for the whole system) field and field acting on the nth particle arises. Indeed, the external field is scattered by other particles before acting on the nth particle. Because distances between the particles can be smaller than the wavelength of the external field, the field acting on the nth particle is generally not a long-wavelength one. This field in general case can change at distances comparable or less than characteristic linear dimension of the particles. It is clear that the property of the particle being not point-like can play an essential role in the formation of the local field inside the system. Here the phrase “short-range” component of the field is used to denote the field component which varies at distances about or less than the linear dimension of the particle. Then, we use the fact that the external field for nth particle is formed by external long-wavelength field and reradiated fields may be taken into account by using the so-called incoming field for nth particle. This field is a sum of the external field acting on the nth particle and fields reradiated by other n-1 particles. The incoming field should be considered as an initial field when calculating the effective susceptibility of the system. This effective susceptibility is that of the nth particle located in a medium consisting of the external space and n-1 particles situated inside it. Then as a first step, one should consider the single-particle problem.

2. The general principles of calculations of the effective susceptibility

To calculate the self-consistent field inside the particle made with material characterized by electrical susceptibility (the linear response on the local field) \( \chi_{ij}(\omega) \), one needs to use the Lippmann–Schwinger equation \[ \tag{1} \]

\[
E_i(\bar{R}, \omega) = E_i^{(0)}(\bar{R}, \omega) + a \int_{V_r} d\bar{R}' \times G_{ij}(\bar{R}, \bar{R}', \omega) \chi_{ij}(\omega) E_j(\bar{R}', \omega),
\]

where integration is over the volume of the particle, \( E_i^{(0)}(\bar{R}, \omega) \) is the external (relative to the particle) field, and (factor) \( a = -k_0^2 / 4\pi \epsilon_0 \), \( k_0 = \omega/c \), with c light velocity. Eq. (1) is written in terms of electrical susceptibility connecting the local polarization with the local (total field) by constitutive equation:

\[
P_i(\bar{R}, \omega) = \chi_{ij}(\omega) E_j(\bar{R}, \omega).
\]

It should be noted that the proposed scheme can work effectively for electrical susceptibility and for conductivity as well. The latter can be both usual electric current conductivity and jump conductivity. In this case Eq. (2) should be rewritten as

\[
J_i(\bar{R}, \omega) = -i\omega \chi_{ij}(\omega) E_j(\bar{R}, \omega) = \sigma_{ij}(\omega) E_j(\bar{R}, \omega)
\]

and factor \( a \) should be replaced with \(-i\omega\mu_0 \) in Eq. (1) and also the susceptibility \( \chi_{ij}(\omega) \) must be replaced with \( \sigma_{ij}(\omega) \). The solution of the above implicit integral equation can be found by several methods. One of them is the method of discretization of the scattering system under consideration [17–19]. As a result, one obtains instead of the integral equation a set of linear algebraic equations:

\[
E_i(\bar{R}_x, \omega) = E_i^{(0)}(\bar{R}_x, \omega) + a \sum_{\beta=1}^{N} G_{ij}(\bar{R}_x, \bar{R}_\beta, \omega) \times \chi_{\beta j}(\bar{R}_\beta, \omega) E_j(\bar{R}_\beta, \omega),
\]

where \( x = 1, 2, \ldots, N \). Here it is assumed that the self-consistent field and the susceptibility are constant inside of each of \( N \) small sub-volumes. The above set of the self-consistent equations can be solved exactly with standard procedures of linear algebra. The problem is that the computing time and the number of discretization elements become rapidly unacceptable for extended structures.

The elegant method of solution of Eq. (1) in the case when the kernel of this integral equation is factorized was proposed and widely used in Refs. [16,20]. This method was used for nonlocal initial
susceptibility written in the form

$$Z_{ij}(\tilde{R}, \tilde{R}', \omega) = \sum_{x, \beta} C_{x\beta}(\omega) j_{x\beta}^{\mu}(\tilde{R}) j_{j}^{\mu}(\tilde{R}') ,$$

(5)

where $j_{x\beta}^{\mu}(\tilde{R})$ is the transition current density for electron transiting from state $x$ to state $\beta$. As a result the Lippmann–Schwinger equation can be rewritten in the form of the system of algebraic equations

$$\gamma_{x\beta}^{\mu} = \gamma_0^{\mu} + \sum_{x', \beta'} C_{x'\beta'}(\omega) N_{x'\beta'}^{\mu} \gamma_{x'\beta'}^{\mu} ,$$

(6)

with

$$\gamma_0^{\mu} = \int_V d\tilde{R} j_{i}^{\mu}(\tilde{R}) E_i(\tilde{R}) ,$$

$$\gamma_0 = \int_V d\tilde{R} j_{i}^{\mu}(\tilde{R}) E_i^{(0)}(\tilde{R}) ,$$

(7)

$$N_{x'\beta'}^{\mu} = a \int_V d\tilde{R} d\tilde{R}' j_{i}^{\mu}(\tilde{R}) G_{i,ik}(\tilde{R}, \tilde{R}') j_{k}^{\mu}(\tilde{R}') .$$

(8)

Solving Eqs. (6) with respect to $\gamma_{x\beta}^{\mu}$, one obtains

$$\gamma_{x\beta}^{\mu} = \tilde{A}_{x\beta} \gamma_0^{\mu} \approx A_{x\beta}^{\mu} E_i^{(0)}$$

(9)

with

$$A_{x\beta}^{\mu} = \tilde{A}_{x\beta} \int_V d\tilde{R} j_{i}^{\mu}(\tilde{R}) .$$

(10)

Since the external field is of long wavelength, here the approximation

$$\int_V d\tilde{R} j_{i}^{\mu}(\tilde{R}) E_i^{(0)}(\tilde{R}') \approx \int_V d\tilde{R} j_{i}^{\mu}(\tilde{R}') \cdot E_i^{(0)}(\tilde{R})$$

(11)

is used. In this expression $\tilde{R}$ is any point inside the system. Then, the self-consisting field at any point of the system is

$$E_i(\tilde{R}) = E_i^{(0)}(\tilde{R}) - a \int_V d\tilde{R} G_{ik}(\tilde{R}, \tilde{R}') X_{jkl}^{(f)}(\tilde{R}') E_i^{(0)}(\tilde{R}')$$

(12)

where the effective susceptibility has the form

$$X_{jkl}^{(f)}(\tilde{R}) = \sum_{x, \beta} C_{x\beta}(\omega) j_{x\beta}^{\mu}(\tilde{R}) A_{i\mu}^{\beta} .$$

(13)

Recently, another method of solution of Eq. (1) was proposed. It is based on the exact summation of the iteration procedure series [21–23]. In the frame of this method the effective susceptibility of the particle can be calculated. The results of this method will be used here. To consider the problem of calculation of the self-consistent field in the system of non-point-like particles we shall use the recently developed scheme for solution of the Lippmann–Schwinger equation. Let a particle of volume $V_1$ characterized by electrical susceptibility of the material $\chi_{jk}^{(1)}(\omega)$ be acted upon by external electrical field $\tilde{E}^{(0)}(\tilde{R}, \omega)$. Then, the self-consistent field in the system is determined by the integral equation

$$E_i(\tilde{R}) = E_i^{(0)}(\tilde{R}) + a \int_{V_1} d\tilde{R}' G_{ij}(\tilde{R}, \tilde{R}') X_{jkl}^{(1)}(\tilde{R}) E_k^{(0)}(\tilde{R}') .$$

(14)

Analytical solution of Eq. (14) can be written via an effective susceptibility (the linear response on the long-range external field $\tilde{E}^{(0)}(\tilde{R}, \omega)$) [21–23]

$$X_{jkl}^{(1)}(\tilde{R}, \omega) = \chi_{jk}^{(1)}(\omega) [\delta_{jk} - S_{jk}^{(1)}(\tilde{R}, \omega)]^{-1} ,$$

(15)

with the self-energy part

$$S_{jl}^{(1)}(\tilde{R}, \omega) = a \int_{V_1} d\tilde{R}' G_{jm}(\tilde{R}, \tilde{R}', \omega) X_{ml}^{(1)}(\omega) .$$

(16)

A variation of the external field at the distance about the linear dimension of the particles was neglected in the integrand of self-energy part [Eq. (16)]. We use similar approximation for calculations of the self-energy parts in this work. Then, the solution of Eq. (14) is presented in the form

$$E_i(\tilde{R}) = E_i^{(0)}(\tilde{R}) + a \int_{V_1} d\tilde{R}' G_{ij}(\tilde{R}, \tilde{R}') \chi_{jkl}^{(1)}(\tilde{R}) E_k^{(0)}(\tilde{R}')$$

(17)

It should be borne in mind that integrals in Eqs. (16) and (17) and similar ones (see equations below) can be improper at $\tilde{R} = \tilde{R}'$ due to singularity of the Green dyadic at $\tilde{R} = \tilde{R}'$. This problem can be dealt with by making use of the regular procedure of excluding a “principal volume” when calculating the integral in Eqs. (16) and (17), and similar ones [24,25].
It should be noted that a similar approach could be formulated for thin film. In this case one can perform the Fourier transformation in the film plane (XOY-plane). Then, the integrals in Eqs. (14)–(17) can be transformed to one-dimensional ones (over coordinate normal to the plane of the film). In the case of ultra-thin film situated at the plane \( z = 0 \), these integrals can be calculated approximately and self-energy part transforms to

\[
\bar{G}_{jm}(\tilde{k}, \omega, z) = \bar{G}^{(1)}_{jm}(\omega) \quad (h \text{ is the thickness of the film}).
\]

Similar result was obtained earlier by another method in Refs. [26,27]. Moreover, as it was shown in Ref. [28], the effective susceptibility for ultra-thin film calculated in Refs. [26,27] fulfills the optical theorem relations.

3. Self-consistent field in a many-body system

In this section we shall discuss the approach for calculation of the effective susceptibility of a many-body system consisting of arbitrary number of nano-particles. To understand the characteristic features of the proposed approach, let us start the consideration for the system consisting of two particles.

3.1. System consisting of two particles

Let us add a second particle to the system consisting of a single particle (number 1). Considering the currents generated inside the second particle one can see that external (relative to this particle) field consists of two parts—the external field and the field reradiated by the particle number 1. Then, we can use the idea of the pseudo-vacuum Green function, i.e., Green function of the system without particle number 2. To calculate the self-consistent field in the system consisting of two particles one can introduce the generalized Green dyadic

\[
G_{ij}^{(2)}(\tilde{R}, \tilde{R}').
\]

The Green dyadic incorporates the indirect scattering channel, i.e., field scattering by the particle number ‘1’. Then, the main integral equation for the self-consistent field can be written in the following simple form:

\[
E_i(\tilde{R}) = E_i^{(0)}(\tilde{R}) + \sum_{j=1} G_{ij}(\tilde{R}, \tilde{R}_1) E_j^{(0)}(\tilde{R}_1) + a \int_{V_1} d\tilde{R}' G^{(2)}_{ij}(\tilde{R}, \tilde{R}') \chi^{(2)}_{kl}(\tilde{R}') \times G_{lk}(\tilde{R}', \tilde{R}).
\]

Here \( \chi^{(2)}_{jl}(\omega) \) is the electrical susceptibility of the material of the ‘2nd’ particle. It should be noted that the main difference between the starting integral equation [Eq. (1)] and the above equation is the presence of the additional Green dyadic [the second term in Eq. (18)]. This Green dyadic takes into account the indirect scattering channel related to the field scattered by the ‘1st’ particle (see Fig. 1). In Eqs. (18) and (19) the designation \( \tilde{R}_1 \) points to the ‘1st’ particle position and the Green dyadic averaged over the ‘1st’ particle volume \( V_1 \) is introduced. In the general case this averaged dyadic connects two Green functions by integral relation

\[
\bar{G}_{ij}(\tilde{R}, \tilde{R}_1) G_{ij}(\tilde{R}_1, \tilde{R}').
\]

Second term in the right side of Eq. (19) describes a part of the incoming (relatively to ‘2nd’ particle) field,
namely, the field scattered by ‘1st’ particle:

\[
G_{ij}(\vec{R}, \vec{R}_1)E_i^{(0)}(\vec{R}_1) = a \int_{V_1} d\vec{R}'' G_{ik}(\vec{R}, \vec{R}'')X_{kl}^{(1)}(\vec{R}'')E_l^{(0)}(\vec{R}'').
\]

(21)

In the case when spatial dependence of the external field is represented as

\[
E_i^{(0)}(\vec{R}) = E_i^{(0)} e^{i\vec{k} \cdot \vec{r}},
\]

(22)

with wave vector \(\vec{k}\), Eq. (21) can be rewritten in the form

\[
G_{il}(\vec{R}, \vec{R}_1) \cdot E_l^{(0)}(\vec{R}_1) = P_{il}^{(1)}(\vec{R}, \vec{R}_1)E_l^{(0)}(\vec{R}),
\]

(23)

with incoming field factor given by

\[
P_{il}^{(1)}(\vec{R}, \vec{R}_1) = a \int_{V_1} d\vec{R}'' G_{ik}(\vec{R}, \vec{R}'')
\times X_{kl}^{(1)}(\vec{R}'')e^{i\vec{k} \cdot (\vec{R}'' - \vec{R})}.
\]

(24)

Then Eq. (19) can be rewritten via illumination operator

\[
I_{ij}^{(2)}(\vec{R}, \omega) = \delta_{ij} + P_{ij}^{(1)}(\vec{R}, \vec{R}_1)
\]

(25)

which allows one to take into account the response of the ‘2nd’ particle on short-range incoming field caused by reradiation of long-wavelength external field by the particle number 1:

\[
E_i(\vec{R}) = I_{ij}^{(2)}(\vec{R})E_j^{(0)}(\vec{R})
+ a \int_{V_2} d\vec{R}' G_{ij}^{(2)}(\vec{R}, \vec{R}')I_{kl}^{(2)}(\vec{R}')E_l^{(0)}(\vec{R}').
\]

(26)

Solution of Eq. (26) can be written via effective susceptibility \(X_{ij}^{(2)}(\vec{R}, \omega)\). In the near-field approximation this solution has a form

\[
E_i(\vec{R}, \omega) = \left[I_{ij}^{(2)}(\vec{R}, \omega) + a \int_{V_2} d\vec{R}' G_{ij}^{(2)}(\vec{R}, \vec{R}')I_{kl}^{(2)}(\vec{R}', \omega)\right]E_j^{(0)}(\vec{R}, \omega)
\times X_{ij}^{(2)}(\vec{R}', \omega)I_{kl}^{(2)}(\vec{R}', \omega).
\]

(27)

The effective susceptibility of the ‘2nd’ particle can be calculated with the method developed in Refs. [21–23]. Then

\[
X_{ij}^{(2)}(\vec{R}, \omega) = \left[S_{ik}^{(2)}(\vec{R}, \omega)\right]^{-1}
\]

(28)

The appearance of illumination operator in the integrand in Eq. (27) is connected with the necessity to calculate the response on the incoming field which consists of both reradiated by ‘1st’ particle field and external field. It should be noted that the incoming field is the external one for ‘2nd’ particle. Because the incoming-field factor depends on the wave vector of harmonic external field [see Eq. (22)], one should remember that the self-consistent field is calculated for the case of harmonic external field characterized by the wave vector \(\vec{k}\) (Fig. 2).

3.2. System consisting of three particles

We have discussed above the principles of calculation of the self-consistent field for systems consisting of one and two particles. However, analysis of these two cases does not allow us to understand the
characteristic features of many-particle interactions. Before developing general approach for a many-particle system, one should consider a system consisting of three particles. To make this, one can introduce the generalized Green dyadic

$$G^{(3)}_{ij}(\vec{R},\vec{R}') = G_{ij}(\vec{R},\vec{R}') + G_{ik}(\vec{R},\vec{R}_2)G^{(2)}_{kj}(\vec{R}_2,\vec{R}')$$

$$+ G_{il}(\vec{R},\vec{R}_1)G^{(2)}_{lj}(\vec{R}_1,\vec{R}').$$

(30)

The Green dyadic incorporates the indirect scattering channel, i.e., field scattering by particles numbered 1 and 2. Indeed substituting Eq. (18) into Eq. (30) one obtains

$$G^{(3)}_{ij}(\vec{R},\vec{R}') = G_{ij}(\vec{R},\vec{R}') + G_{ik}(\vec{R},\vec{R}_1)G_{lj}(\vec{R}_1,\vec{R}')$$

$$+ G_{ik}(\vec{R},\vec{R}_2)G_{lj}(\vec{R}_2,\vec{R}')$$

$$+ G_{il}(\vec{R},\vec{R}_1)G_{jk}(\vec{R}_1,\vec{R}_2)G_{lj}(\vec{R}_2,\vec{R}')$$

$$+ G_{ik}(\vec{R},\vec{R}_2)G_{lj}(\vec{R},\vec{R}_1)G_{lj}(\vec{R},\vec{R}_1,\vec{R}').$$

(31)

that indicates four different channels of scattering in the system (see also Fig. 3). It should be noted that in Eq. (31) one needs to use the Green functions product without repetition of the indexes in the arguments $\vec{R}$, inside each Green function. Taking some steps analogous to the previous case, one can write the main integral equation for the self-consistent field in the form

$$E_i(\vec{R}) = l^{(3)}_{ij}(\vec{R})E^{(0)}_j(\vec{R})$$

$$+ a \int_{V_3} d\vec{R}' G^{(3)}_{ij}(\vec{R},\vec{R}')\chi^{(3)}_{kl}E_k(\vec{R}').$$

(32)

Here $\chi^{(3)}_{kl}(\omega)$ is the electrical susceptibility of the material of the 3rd particle (the linear response on the local field),

$$l^{(3)}_{ij}(\vec{R}) = \delta_{ij} + P^{(2)}_{ij}(\vec{R},\vec{R}_1) + P^{(2)}_{ij}(\vec{R},\vec{R}_2)$$

(33)

is the illumination operator, which allows us to take into account that the external field for 3rd particle is the field consisting of long-wavelength external field [Eq. (18)] and fields reradiated by the system consisting of ‘1st’ and ‘2nd’ particles. This field, generally speaking, is not the long-range one. In Eq. (33) the designation

$$P^{(2)}_{ij}(\vec{R},\vec{R}_i) = a \int_{V_3} d\vec{R}'' G^{(2)}_{ik}(\vec{R},\vec{R}'')$$

$$\times \chi^{(2)}_{kl}(\vec{R}'') e^{i\vec{k} \cdot (\vec{R}' - \vec{R})}$$

(34)

is introduced. It should be noted that the main idea of this approach is very close to the idea of pseudo-vacuum Green function which was discussed in detail by Keller [16]. One can see that the main difference between the starting integral equation [Eq. (1)] and the above equation is the presence of the generalized Green dyadic $G^{(3)}_{ij}(\vec{R},\vec{R}')$ accounting for the indirect scattering channel related to the field scattering by the system of the particles 1 and 2 (see Fig. 3). As before, one can introduce the effective susceptibility of the 3rd particle

$$\chi^{(3)}_{kl}(\vec{R}, \omega) = \chi^{(3)}_{kl}(\omega)[\delta_{ik} - S^{(3)}_{lk}(\vec{R}, \omega)]^{-1},$$

(35)

with the self-energy part

$$S^{(3)}_{lk}(\vec{R}, \omega) = a \int_{V_3} d\vec{R}' G^{(3)}_{im}(\vec{R},\vec{R}', \omega)\chi^{(3)}_{mk}(\omega).$$

(36)

Then, in the case of monochromatic external field, the self-consistent field at any point in the system has the form

$$E_i(\vec{R}, \omega) = \left[ l^{(3)}_{ij}(\vec{R}, \omega) + a \int_{V_3} d\vec{R}' G^{(3)}_{ij}(\vec{R},\vec{R}', \omega)$$

$$\times \chi^{(3)}_{kl}(\vec{R}, \omega)l^{(3)}_{mj}(\vec{R}', \omega)\right] E^{(0)}_j(\vec{R}, \omega).$$

(37)

One should note that operator $l^{(3)}_{ij}(\vec{R}, \omega)$ describes the linear response of the 3rd particle on the strongly inhomogeneous field caused by reradiation processes in
the system consisting of two particles. As can be seen from Eq. (33), the illumination operator is defined via its action on the long-wavelength external field (Fig. 3).

3.3. System consisting of an arbitrary number of particles

To calculate the self-consistent field in the system consisting of arbitrary number of particles one can introduce the generalized Green dyadic via recurrent formula

\[
G^{(n)}_{ij}(\vec{R}, \vec{R'}) = G_{ij}(\vec{R}, \vec{R'}) + \sum_{a}^{n-1} G_{il}(\vec{R}, \vec{R}_a)G^{(n-1)}_{lj}(\vec{R}_a, \vec{R'}) + \sum_{a}^{n-1} \sum_{b \neq a}^{n-1} G_{ik}(\vec{R}, \vec{R}_a)G_{kl}(\vec{R}_a, \vec{R}_b)G_{bj}(\vec{R}_b, \vec{R'}) + \cdots
\]

The Green dyadic in this general case incorporates all indirect scattering channels, i.e., field scattering by the particles from number 1 up to number \( n - 1 \). The sign \( * \) in this equation means that only products where pairs \( \ldots \vec{R}_p \) \( G_{ij}(\vec{R}_p) \ldots \) with different \( p \) are presented only once. As a result, Eq. (38) can be rewritten via the Green function of the space in which the particles are situated:

\[
G^{(n)}_{ij}(\vec{R}, \vec{R'}) = G_{ij}(\vec{R}, \vec{R'}) + \sum_{a}^{n-1} G_{ik}(\vec{R}, \vec{R}_a)G_{lj}(\vec{R}_a, \vec{R'}) + \sum_{a}^{n-1} \sum_{b \neq a}^{n-1} G_{ik}(\vec{R}, \vec{R}_a)G_{kl}(\vec{R}_a, \vec{R}_b)G_{bj}(\vec{R}_b, \vec{R'}) + \cdots
\]
This equation indicates \( n \) different channels of scattering in the system (see also Fig. 4). The self-consistent field at the arbitrary point inside the system consisting of \( n \) particles obeys the equation

\[
E_i(\vec{R}_n, \omega) = \sum_{j=1}^{n} \int d\vec{R}' G_{ij}^{(n)}(\vec{R}_n, \vec{R}', \omega) \times X_{jl}^{(n)}(\omega) E_{ij}(\vec{R}', \omega). 
\]  

(40)

Here \( X_{jl}^{(n)}(\omega) \) is the electrical susceptibility of the material of the \( n \)th particle (the linear response on the local field), and

\[
E_i^{(j)}(\vec{R}, \omega) = I_{ij}^{(n)}(\vec{R}, \omega) E_j^{(0)}(\vec{R}, \omega)
\]

\[
= E_i^{(0)}(\vec{R}, \omega) + \sum_{a}^{n-1} G_{ij}^{(n-1)}(\vec{R}, \vec{R}_a, \omega) \times E_j^{(0)}(\vec{R}_a, \omega).
\]  

(41)

is the incoming field determined by incoming operator \( I_{ij}^{(n)} \). Eq. (40) means that the local field is formed by effective scattering between subsystem consisting of \( n - 1 \) particles and particle of number \( n \). Analogously to previous cases one introduces the effective susceptibility of the \( n \)th particle:

\[
X_{jl}^{(n)}(\vec{R}, \omega) = Z_{jl}^{(n)}(\omega)[\delta_{lk} - S_{lk}^{(n)}(\vec{R}, \omega)]^{-1},
\]  

(42)

with the self-energy part

\[
S_{lk}^{(n)}(\vec{R}, \omega) = a \int d\vec{R}' G_{lm}^{(n)}(\vec{R}, \vec{R}', \omega) X_{mk}^{(n)}(\omega).
\]  

(43)

One should note once again that the multi-scattering processes are taking into account in the effective susceptibility of the \( n \)th particle via the use of the pseudo-vacuum Green function which describes the photon propagation inside the system consisting of \( n - 1 \) particles. The solution of Eq. (40) for long-wavelength monochromatic external field characterized by wave vector \( \vec{k} \) [see Eq. (22)] gives the self-consistent field at any point inside the system

\[
E_i(\vec{R}, \omega) = L_{ij}^{(n)}(\vec{R}, \omega) E_j^{(0)}(\vec{R}, \omega),
\]
with
\[ L_{ij}^{(n)}(\tilde{R}, \omega) = I_{ij}^{(n)}(\tilde{R}, \omega) + a \int_{V_2} d\tilde{R}' G_{ij}^{(n)}(\tilde{R}, \tilde{R}', \omega) \]
\[ \times X_{lm}^{(n)}(\tilde{R}', \omega) I_{mj}^{(n)}(\tilde{R}', \omega) e^{i \tilde{k} \cdot (\tilde{R} - \tilde{R}')}. \]  
(44)

One should note that the general solution of the many-body problem in nonlocal electrodynamics of a system of meso-particles is a complicated recurrent formula. But this way is well adapted for numerical calculations because any recurrent approach can be rather easily realized numerically.

4. Susceptibility of the many-body system

To understand the general feature of the problem of determination of susceptibility of a many-body system let us imagine an object consisting of two identical parts. Then the effective susceptibility of the object can be evidently represented in the form
\[ X_{ij}^{(1)}(\tilde{R}, \omega) = [\chi_{ij}(\omega)]^{-1} \]
\[ - a \int_{V_1 + V_2} d\tilde{R}' G_{ij}(\tilde{R}, \tilde{R}', \omega) \]  
(45)

Let the object be cut off into two identical parts and let those be pressed to each other (see Fig. 5). In the case where surface effects are neglected the effective susceptibility of this system can be written in the form of Eq. (45). The situation becomes different from previous cases if one disconnects the parts of the object. Indeed, in this case the external field for particle number ’2’ consists of the external field and the field reradiated by the particle 1 (see Fig. 6). To calculate the effective susceptibility for particle 2 one needs to use the pseudo-vacuum Green function taking into account all scattering processes in the system “particle 1—medium”. As a result one obtains
\[ X_{ij}^{(2)}(\tilde{R}, \omega) = \chi_{im}(\omega) [\delta_{jm} - a \int_{V_2} d\tilde{R}' G_{ij}(\tilde{R}, \tilde{R}', \omega) \chi_{in}(\omega)]^{-1} \]
(46)

To understand the difference between susceptibilities given by Eqs. (45) and (46), let us substitute

\[ G_{il}^{(2)}(\tilde{R}, \tilde{R}') \] from Eq. (18) in the integrand of Eq. (46). Then, one has
\[ X_{ij}^{(2)}(\tilde{R}, \omega) \]
\[ = \chi_{im}(\omega) [\delta_{jm} - a \int_{V_2} d\tilde{R}' G_{il}(\tilde{R}, \tilde{R}', \omega) \chi_{in}(\omega) \]
\[ - a \int_{V_2} d\tilde{R}' a \int_{V_1} d\tilde{R}'' G_{jk}(\tilde{R}, \tilde{R}'', \omega) \]
\[ \times X_{kn}^{(1)}(\tilde{R}'', \omega) G_{nl}(\tilde{R}'', \tilde{R}', \omega) \chi_{lm}(\omega)]^{-1}. \]  
(47)

It should be noted that using the generalized Green dyadic instead of the simple Green dyadic, which was used in previous case is the characteristic property of the problem of two-body (many-body) system. The second term of the self-energy part of Eq. (47) describes the interaction between the particles. In this connection one can say that Eq. (46) describes the effective susceptibility of the ‘2nd’ particle taking into account all scattering processes in the system. Hence
one can say that the susceptibility means the effective susceptibility of the system. The difference between Eqs. (45) and (46) allows us to state that the use of the approximation

\[
\mathcal{M}_{ij}(\tilde{R},!; 1)
= \begin{cases} 
\mathcal{M}_{ij}(\tilde{R},!; 1) & \text{if } \tilde{R} \in \text{volume of the 1st particle} \\
\mathcal{M}_{ij}(\tilde{R},!; 2) & \text{if } \tilde{R} \in \text{volume of the 2nd particle} \\
\vdots \\
\mathcal{M}_{ij}(\tilde{R},!; n) & \text{if } \tilde{R} \in \text{volume of the } n\text{th particle}
\end{cases}
\]  
(48)

for the study of the electrodynamic properties of a system consisting of \( n \) particles and the consideration of the system as a system formally consisting of a single multi-connected object is not quite correct. Nevertheless, one should note that there are situations when the presentation of the system as multi-connected can be supposed correct. It could be, for example, the object situated at the surface of a substrate or an object with holes or a porous particle.

Finally, one should emphasize that the main idea of the proposed consideration lies in taking into account the fact that external (relatively the whole system) field and external (relatively the particle under consideration) one are different. The first is controlled by experimentalists and usually is long-wavelength one. The second field, in general, is a short-range one and is not controlled by ones. Moreover, this field should be excluded in the final equations of any theory.

5. Sum rule

In this section, we shall obtain the general relations between susceptibilities of a many-body system. Let the effective susceptibility of the system consisting of \( n \) identical particles be known and given by

\[
X^{(n)}_{ij}(\tilde{R},!; n)
= \left\{ [X_{ij}(\omega)]^{-1} - a \int_V d\tilde{R}' G^{(n)}_{ij}(\tilde{R},\tilde{R}',\omega) \right\}^{-1}. 
\]  
(49)

Let a particle be added to this system. Then, the susceptibility of the new system is

\[
X^{(n+1)}_{ij}(\tilde{R},!; n+1)
= \left\{ [X_{ij}(\omega)]^{-1} - a \int_V d\tilde{R}' G^{(n+1)}_{ij}(\tilde{R},\tilde{R}',\omega) \right\}^{-1}. 
\]  
(50)

Using Eq. (49) one can write

\[
a \int_V d\tilde{R}' G^{(n)}_{ij}(\tilde{R},\tilde{R}',\omega)
= [X_{ij}(\omega)]^{-1} - [X^{(n)}_{ij}(\tilde{R},\omega)]^{-1}. 
\]  
(51)

On the other hand, let us substitute Eq. (38) into the integrand of Eq. (50). As a result, one can rewrite Eq. (50) in the form

\[
X^{(n+1)}_{ij}(\tilde{R},!; n+1)
= \left\{ [X_{ij}(\omega)]^{-1} - a \sum_x G_{ik}(\tilde{R},\tilde{R}_x) \int_V d\tilde{R}' G^{(n)}_{kj}(\tilde{R}_x,\tilde{R}',\omega) \right\}^{-1}. 
\]  
(52)

It should be noted that the inverse effective susceptibility of the single particle is connected with the inverse linear response on the total field via

\[
[X_{ij}(\omega)]^{-1} - a \int_V d\tilde{R}' G_{ij}(\tilde{R},\tilde{R}',\omega)
= [X^{(1)}_{ij}(\tilde{R},\omega)]^{-1}. 
\]  
(53)

Then, Eq. (52) can be rewritten in the form

\[
X^{(n+1)}_{ij}(\tilde{R},!; n+1)
= \left\{ [X^{(1)}_{ij}(\tilde{R},\omega)]^{-1} - a \sum_x G_{ik}(\tilde{R},\tilde{R}_x) [X^{(1)}_{ij}(\tilde{R}_x,\omega)]^{-1} \right\}^{-1}. 
\]  
(54)
which can be reduced to the general expression playing the role of the sum rule:

\[
[X^{(n+1)}_{ji}(\vec{R}, \omega)]^{-1} = \sum_{x} G_{ik}(\vec{R}, \vec{R}_x) [X^{(n)}_{jk}(\vec{R}_x, \omega)]^{-1} - \sum_{x} G_{ik}(\vec{R}, \vec{R}_x) \chi_{ik}(\omega)^{-1}.
\]

(55)

One needs to note that this general equation is true for any number \( n \) \((n > 1)\) of the particles in the system. Let the system under consideration consist of \( n \) particles and \( n \gg 1 \). Let one particle be added to the system. Obviously, macroscopic optical properties of the new system will differ from the properties of the \( n \)-particles system but slightly. It means that one can put \( X^{(n+1)}_{ji}(\vec{R}, \omega) \approx X^{(n)}_{ji}(\vec{R}, \omega) \), and Eq. (55) can be considered as the equation for calculation of the effective susceptibility of the system. The obtained equation is the integral one but for evaluation of the effective susceptibility one can suppose that \( \vec{R}_0 \) is the coordinate of the center of “\( x \)th” particle. Then, the integral equation reduces to the system of algebraic equations for the effective susceptibility of the system. It is clear that this equation can be used to calculate \( X^{(n)}_{ij}(\vec{R}, \omega) \) for a system of the particles forming a regular structure. Indeed, let us consider the system consisting of \( n \) \((n \gg 1)\) identical particles with material characterized by scalar susceptibility \( \chi(\omega) \). Let us introduce the designations

\[
[X^{(n)}_{ji}(\vec{R}, \omega)]^{-1} = \Xi^{(n)}_{ij}(\vec{R}, \omega),
\]

(56)

\[
M^{(n)}_{ij}(\vec{R}, \omega) = [X^{(1)}_{ji}(\vec{R}, \omega)]^{-1}
\]

\[
- \sum_{x} G_{ik}(\vec{R}, \vec{R}_x, \omega) Q_{kj}(\vec{R}_x, \omega),
\]

(57)

where

\[
Q_{kj}(\vec{R}, \omega) = \left[ \delta_{jk} - a \chi(\omega) \int d\vec{R} G_{jk}(\vec{R}, \vec{R}, \omega) \right]^{-1}.
\]

(58)

Then, the equation for evaluation of the effective susceptibility is

\[
\Xi^{(n)}_{ij}(\vec{R}_0, \omega)
\]

\[
- \sum_{x} G_{jk}(\vec{R}_0, \vec{R}_x) \chi^{(1)}_{km}(\vec{R}_x, \omega) \Xi^{(n)}_{ml}(\vec{R}_x, \omega)
\]

\[
= M^{(n)}_{ij}(\vec{R}_0, \omega).
\]

(59)

This equation can be solved by standard algebraic methods. It should be noted that this equation is more simple than that obtained as a result of the accurate procedure for calculation of the effective local susceptibility of the \( n \) particles system described in Section 3.3. The term the effective local susceptibility is used here for the effective susceptibility calculated at the geometrical center of the particle.

6. What does the resonance interaction inside a many-body system mean?

Let us consider an infinite homogeneous medium characterized by constitutive equation

\[
J_{ij}(\vec{K}, \omega) = \chi_{ij}(\vec{K}, \omega) E_{ij}(\vec{K}, \omega).
\]

(60)

The pole part of the susceptibility \( \chi_{ij}(\vec{K}, \omega) \) determines the resonant condition (dispersion relation) which connects the frequency with the wave vector of the external field, providing the maximum value of the field generated by the current [Eq. (60)]. What can one say about absorption by the system in this case? As is well known, the energy of the external field \( E(\vec{K}, \omega) e_i \), absorbed by the system is proportional to

\[
Q = \text{Im} \chi_{ij}(\vec{K}, \omega) e_i e_j,
\]

(61)

with \( e_i \) the unit vector of external field polarization. We see that the field and absorption resonance conditions for uniform homogeneous system coincide.

To analyse the absorption of energy of the external (for the particle) field by mesoparticle let us consider the dissipative function \([29,30]\) which is the quantity of energy dissipated by the particle per unit of time,

\[
Q = \langle (\vec{J} + \vec{J}^*) (\vec{E} - \vec{E}^*) \rangle.
\]

(62)
Here the symbols \( \langle \cdot \cdot \cdot \rangle \) and \( \bar{\cdots} \) mean the averaging over particle volume and time, respectively. \( \hat{E} \) is the local field. After time averaging Eq. (62) transforms to

\[
Q = \langle J_i E_i^* + J_i^* E_i \rangle. \tag{63}
\]

The current induced at arbitrary point inside the particle is connected with the external field via equation

\[
J_i(\bar{R}, \omega) = -i\omega \mu_0 X_{ij}(\bar{R}, \omega) E_j^{(0)}(\bar{R}, \omega). \tag{64}
\]

Because the local field and external field are connected by the local field factor \( L_{ij}(\bar{R}, \omega) \),

\[
E_i(\bar{R}, \omega) = L_{ij}(\bar{R}, \omega) E_j^{(0)}(\bar{R}, \omega), \tag{65}
\]

Eq. (63) can be rewritten in the form (cf. Ref. [30])

\[
Q = \frac{a}{V} \int_V d\bar{R} [X_{ij}(\bar{R}, \omega) L_{ik}^*(\bar{R}, \omega)] E_j^{(0)}(E_k^{(0)})^*. \tag{66}
\]

It should be noted that only amplitude of the external field defines the energy absorption. The expression [Eq. (66)] shows us that anomalous strong absorption of the energy by the particle can be observed as peaks in the frequency dependence of the dissipative function. This situation can naturally be named as a resonance. It should be noted that instead of the pole part of \( \text{Im} L_{ij}(\bar{K}, \omega) e_j e_i \) in the case of bulk sample (infinite system), the resonant dissipation of the monochromatic field by the particle is determined by the pole part of the function

\[
Q(\omega) = \text{Im} \left\{ \frac{a}{V} \int_V d\bar{R} X_{ij}(\bar{R}, \omega) L_{ik}^*(\bar{R}, \omega) e_j e_k \right\}. \tag{67}
\]

A similar situation occurs in the case of the many-body system. One should only change the effective susceptibility and local field factor in Eq. (67) by those for \( n \)-particle system. It should be noted that in the case of the long-wavelength monochromatic external field

\[
E_j^{(0)}(\bar{R}, \omega) = E_j^{(0)} e^{i\omega \tau - ik \bar{R}}, \tag{68}
\]

the local field factor for \( n \)-particles system has the form (44). The absorption of the external field energy by the system is described by the function

\[
Q^{(n)}(\omega) = \text{Im} \left\{ \frac{1}{V} \int_V \int_{\bar{R}_n} d\bar{R} \chi^{(n)}_{ij}(\bar{R}, \omega) \right. \\
\times \left[ L_{ik}^{(n)}(\bar{R}, \omega) \right] e_j e_k \right\}. \tag{69}
\]

The frequencies \( \omega_i \) at which \( Q^{(n)}(\omega) \) has the peaks, should be considered as resonant frequencies. Consequently the energy absorbed by the system is

\[
\Omega(\omega) = a \sum_{i=1}^n Q^{(n)}(\omega)|E^{(0)}|^2. \tag{70}
\]

It is clear that both the effective susceptibility and local field factor involve information about the structure of the system. One can state that configuration resonances [15] of a system of nonpoint particles could be described in the frame of this scheme. Clearly that the field resonances are not directly connected with resonant absorption of the external field energy. Indeed, let the local field at fixed frequency at any \( \bar{R}_m \) point inside the system

\[
E_i(\bar{R}_m, \omega) = L_{ij}^{(n)}(\bar{R}_m, \omega) E_j^{(0)}(\bar{R}_m, \omega) \tag{71}
\]

be anomalously strong at infinitesimal external field. This is possible only when the local field factor is infinitely large. In other words, this can be only when \( L_{ij}^{(n)}(\bar{R}_m, \omega) \) has a pole. It is clear that this condition does not play an essential role in forming the resonant absorption. Indeed, this pole defines only one (or several) integral in the sum of \( n \) integrals in Eq. (70). Nevertheless, this situation can be considered as a local resonance. This resonance is a characteristic of the interactions inside the system. One should note once again that the local resonances do not characterize the resonance absorption of energy of external field by the whole system.

7. Implementation to the two dielectric spheres

To demonstrate the developed method here we calculate the effective susceptibility of the sphere being under the action of the homogeneous external field and discuss the interaction between two spheres. Let the sphere be characterized by a scalar linear response function \( \chi^{(0)}(\omega) \) connected with the dielectric constant
via \( \chi^{(i)} = \epsilon^{(i)} - 1 \) and located in the free space with the Green dyadic \( (R = |\vec{R} - \vec{R}'|) \)

\[
\vec{G} (\vec{R}, \vec{R}', \omega) = \frac{1}{4\pi} \left[ \left( \frac{1}{R} - \frac{i\epsilon}{\omega \cdot R^2} + \frac{c^2}{\omega^2 R^4} \right) \vec{U} + \left( \frac{1}{R} + \frac{3i\epsilon}{\omega \cdot R^2} - \frac{3c^2}{\omega^2 R^4} \right) \vec{e}_\parallel \vec{e}_\parallel \right] e^{ik_0 R}. \tag{72}
\]

Let the spherical particle be subjected to the homogeneous electric field. Substituting Eq. (72) into Eq. (9), one has (similar results were obtained in Refs. [24,25])

\[
S_{jl}^{(1)} (\vec{R}, \omega) = \frac{\chi^{(1)}}{3} \delta_{jl}. \tag{73}
\]

As a result, the effective susceptibility of the sphere in this special case (the sphere is acted by the homogeneous field) can be written as

\[
X_{(0)}^{(1)} = \frac{\chi^{(1)}}{1 + \chi^{(1)} / 3}. \tag{74}
\]

Using Eq. (74), one obtains the well known result that the field inside the sphere is homogeneous and connected with the external field via Eq. [29]

\[
E_i (\vec{R}, \omega) = \left[ 1 - \frac{\chi^{(1)}}{3} \frac{1}{1 + \chi^{(1)}/3} \right] E_i^{(0)} (\omega) = \left[ \frac{1}{1 + (\epsilon^{(1)} - 1)/3} \right] E_i^{(0)} (\omega) = \frac{3}{\epsilon^{(1)} + 2} E_i^{(0)} (\omega). \tag{75}
\]

To consider an interaction between two spheres, it is convenient to choose the coordinate system, in which origin coincides with the center of the sphere 2 and the z-axis points toward sphere 1 (Fig. 7). Let radii of ‘1st’ and ‘2nd’ spheres are equal to \( r_0 \) and \( R_0 \), respectively. We have to find the self-consistent field at the site of sphere 1 as a function of the distance \( R \) between spheres. At first, we consider sufficiently small spheres and assume that the distance between the particles is so large that one can suppose those to be situated in the homogeneous fields. One can then obtain for the self-consistent field at the site of sphere 1 the following relation:

\[
E_i (\vec{R}_1) = \left\{ 1 + \frac{F^{(2)}(\vec{R}_1, \vec{R}_2)}{1 + \chi^{(2)}/3 - F^{(2)}(\vec{R}_1, \vec{R}_2)F^{(1)}(\vec{R}_2, \vec{R}_1)} \right\} E_i^{(0)} (\vec{R}_1), \tag{76}
\]

where the following designations were used

\[
F^{(1)}(\vec{R}_2, \vec{R}_1) = \frac{X_{(0)}^{(1)}}{3} \rho_0 G_i^{(d)} (\vec{R}_2, \vec{R}_1), \tag{77}
\]

\[
F^{(2)}(\vec{R}_1, \vec{R}_2) = \frac{\chi^{(2)}}{3} \rho_0^{(2)} G_i^{(d)} (\vec{R}_1, \vec{R}_2). \tag{78}
\]

Here \( \vec{R}_1 \) and \( \vec{R}_2 \) are coordinates of the centers of the 1st and 2nd spherical particles, respectively, and the following notation is used:

\[
G_i^{(d)} (\vec{R}_2, \vec{R}_1) = G_i^{(d)} (\vec{R}_1, \vec{R}_2) = \begin{cases} \frac{1}{R^3}, & i = x, y, \\ \frac{2}{R^3}, & i = z. \end{cases} \tag{79}
\]

Let us introduce the normalized scattering parameters for both spheres

\[
\rho_1 = \alpha_1 / (4\pi\epsilon_0 R^3) \quad \text{and} \quad \rho_2 = \alpha_2 / (4\pi\epsilon_0 R^3), \tag{80}
\]
where
\[ x_1 = X_{(1)}^{(1)} \varepsilon_0 \frac{4\pi r_0^3}{3} \quad \text{and} \quad x_2 = X_{(1)}^{(2)} \varepsilon_0 \frac{4\pi R_0^3}{3}, \quad (81) \]

with
\[ X_{(0)}^{(2)} = \frac{\chi^{(2)}}{1 + \chi^{(2)}/3} \quad (82) \]

the effective susceptibility of the ‘2nd’ particle in the homogeneous field. The above relation [Eq. (76)] can be rewritten in the form
\[ E_{x,y}(\vec{R}_p) = \left[ \frac{1 - \rho_2}{1 - \rho_1 \rho_2} \right] E_{x,y}^{(0)}(\vec{R}_p), \]
\[ E_z(\vec{R}_p) = \left[ \frac{1 + 2\rho_2}{1 - 4\rho_1 \rho_2} \right] E_z^{(0)}(\vec{R}_p). \quad (83) \]

These relations coincide with those obtained in the point-dipole approximation [31]. One should note that expressions (83) were calculated in terms of the self-field [Eq. (76)], but under the assumption that the external (for each of the two spheres) field varies slowly. It is clear that the latter approximation can be justified only when the particle separation is large enough in comparison with particle sizes. For sufficient small separations, the field scattered by one of two particles results in an inhomogeneous field acting on another particle. This circumstance has to be borne in mind, and the appropriate corrections have to be implemented when calculating the self-consistent field, especially for strongly interacting particles.

The expressions for sphere polarizabilities [Eqs. (81)] can be rewritten via the dielectric constants. One obtains the equations having the form of the Lorentz–Lorenz relation:
\[ x_1 = 4\pi \varepsilon_0 r_0^3 \frac{\chi^{(1)}}{3 + \chi^{(1)}} = 4\pi \varepsilon_0 r_0^3 \frac{\varepsilon_1 - 1}{\varepsilon_1 + 2}, \quad (84) \]
\[ x_2 = 4\pi \varepsilon_0 R_0^3 \frac{\chi^{(2)}}{3 + \chi^{(2)}} = 4\pi \varepsilon_0 R_0^3 \frac{\varepsilon_2 - 1}{\varepsilon_2 + 2}. \quad (85) \]

Thus, we have demonstrated that in the frame of developed approach the well known result for the polarizability of the small spheres in the homogeneous field (Lorentz–Lorenz formula) is obtained automatically for interaction between two widely separated spheres. Resonance interaction between two spherical particles was considered in detail in Ref. [32]

8. Concluding remarks

Analysis of electrodynamical interactions between small particles shows that the possibility exists to develop the scheme for explicit calculation of the effective susceptibility for a many-body system. The definition of external (for the particle under consideration) field is the main point of the developed approach. The main idea is based on the concept of the pseudo-vacuum Green function. The particle is considered to be in a medium. It is supposed that the electrodynamical Green function of the medium (system without the particle under consideration) is known. Consequently considering the systems consisting of one, two, etc. particles one can obtain the recurrent relation connecting the susceptibilities of the n-particle system and the n + 1-particle one. The proposed scheme can be used for the problem when the nonpointness and nonlocalities play an essential role. It can be, for example, the problem of describing the scanning near-field optical microscopy, the problem of interaction between molecular layer and substrate, etc. As an example, we have calculated the interaction between two spheres in the frame of the approach developed. The well-known results were obtained, namely the Lorentz–Lorenz formula for the effective susceptibility of the spherical particle in homogeneous field was obtained after calculation of the self-consistent field at the particle when the particles were situated at rather large distances from one another. The scheme developed in the present work can be useful for calculation of the electrodynamical effects in various small-particle systems. For example, one should note that the nonpointness of the particle leads to effective renormalization of the energy levels for quantum dots. Taking into account the field–field corrections inside the quantum dots gives the Lamb shift of the levels (see, for example, Refs. [12,13,33–36]). Especially this shift can be essential (up to 10%) when the quantum dot is situated under a surface [36] (the so-called hidden quantum dot). It would be of interest to study the self-field action in the whole hidden layer of the quantum dots. One can believe that the dense layer has to be characterized by strong local field effects. To calculate those one could use the approach proposed in this work. Other systems, for which the proposed approach could be useful, are the composites. As examples of such
systems one can point the molecular nano-composites with high density of the molecular clusters [37] and complexes of silica spheres [38].

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References