Physicist in the kitchen: exploring the gastronomic universe

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in collaboration with
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General formula for the cooking time

Heat flow

\[
\frac{\partial T(\rho, \tau)}{\partial \tau} = \kappa \Delta T(\rho, \tau) = \kappa \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left( \rho^2 \frac{\partial T(\rho, \tau)}{\partial \rho} \right)
\]

\[
\begin{bmatrix}
\frac{1}{\tau} \\
\frac{\kappa}{r^2}
\end{bmatrix} = \begin{bmatrix}
1 \\
r^2
\end{bmatrix}
\]

\[\tau_{cook} = aD^2 + b\]
1. Soft-boiled egg

Dr. Charles D. H. Williams, a lecturer in physics at University of Exeter:

\[ \tau_{cook} = 0.451 M^{2/3} \ln \left( \frac{T_{egg} - T_{water}}{T_{yolk} - T_{water}} \right) \]
2. The Thanksgiving day turkey secret

\[ \tau_{\text{cook}} = aM^{2/3} + b \]

<table>
<thead>
<tr>
<th>Mass (pounds)</th>
<th>Cooking time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>3.5</td>
</tr>
<tr>
<td>12</td>
<td>4.5</td>
</tr>
<tr>
<td>16</td>
<td>5.5</td>
</tr>
<tr>
<td>20</td>
<td>6.5</td>
</tr>
<tr>
<td>24</td>
<td>7.35</td>
</tr>
</tbody>
</table>
3. Physics of Pasta

Het flow and water diffusion

Heat transfer:

\[ \frac{\partial T}{\partial \tau} = \kappa \Delta T \]

Water diffusion:

\[ \frac{\partial n}{\partial \tau} = D \Delta n \]

Cooking time: \[ \tau = aD^2 + b \]
### Table of cooking times for the different types of cylindrical shape pasta

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capellini n.1</td>
<td>1.15 mm</td>
<td>--</td>
<td>3 min</td>
<td>2 min</td>
</tr>
<tr>
<td>Spaghettini n. 3</td>
<td>1.45 mm</td>
<td>--</td>
<td>5 min</td>
<td>5.0 min</td>
</tr>
<tr>
<td>Spaghetti n. 5</td>
<td>1.75 mm</td>
<td>--</td>
<td>8 min</td>
<td>8.2 min</td>
</tr>
<tr>
<td>Vermicelli n. 7</td>
<td>1.90 mm</td>
<td>--</td>
<td>11 min</td>
<td>10.7 min</td>
</tr>
<tr>
<td>Vermicelli n. 8</td>
<td>2.05 mm</td>
<td>--</td>
<td>13 min</td>
<td>13.0 min</td>
</tr>
<tr>
<td>Bucatini</td>
<td>2.70 mm</td>
<td>1 mm</td>
<td>8 min</td>
<td>25 min ?!</td>
</tr>
</tbody>
</table>

\[ \tau = aD^2 + b \]

\[ a = 3.8 \text{ min} / \text{mm}^2 \]

\[ b = -3 \text{ min} \]
The case of bucatini requires the particular treatment. The correct formula is:

\[ \tau = a(D-d)^2 + b \]

\[ \rho \frac{gh}{2\sigma/r_{\text{min}}} \]

\[ d_{\text{min}} = 2r_{\text{min}} \sim 4\sigma/\rho gh \]

\[ h = 2\text{cm}, \sigma = 0.05 \text{ N/m} \]

\[ g = 9.8 \text{ m/s}^2 \]

\[ d_{\text{min}} \sim 1\text{mm} \]

For bucatini: \[ \tau = 8.2 \text{ min} ! \]
Fall down of our simple theory for too thin pasta

Capellini: $D = 1.15 \text{ mm}$

\[ \tau_{\text{cap}} = 3.8 \left[ 1.15 \right]^2 - 3 \approx 2 \text{ min} \]

\[ aD_{cr}^2 + b = 0 \]

\[ D_{cr} = \sqrt{|b|/a} \approx 0.85 \text{ mm} \]
Why spaghetti do not break into two pieces?
The breaking dynamics:
(B.Audoli, S.Neukirch, PRL, August 2005)

\[ 4u''''(\xi) + \xi^2 u''(\xi) + 3\xi u'(\xi) = 0. \]

\[ \xi = \frac{s/L}{\sqrt{t/T}} = \frac{s}{\sqrt{\gamma t}}. \]

**FIG. 1.** The dynamics of a rod fragment following the initial breaking event in a brittle rod is modeled by releasing at time \( t = 0 \) a rod with fixed length \( L \), initial curvature \( \kappa_0 \), and no initial velocity.
When the rod is broken, one could expect that the generated elastic waves help to both pieces of rod to relax to equilibrium.

Invece, si verifica che queste onde aumentano lo stress locale, rendendo probabile la rottura della bacchetta anche in altri punti. Ne consegue una frammentazione.

FIG. 3. A dry spaghetti is bent into an arc of circle and suddenly set free, while its lower end remains clamped. Its subsequent dynamics exhibits a local increase of curvature. Selected frames shot with a fast camera at 1000 Hz: (a) release $t_a = 0$, (b) intermediate frame $t_b = 0.0159T$, (c) frame just before rupture $t_c = 0.0509T$, and (d) frame after rupture $t_d = 0.0596T$. Predictions of the self-similar and numerical simulations based on Eq. (1) are superimposed, without any adjustable parameters: rod profile (dotted line) and osculating circle (dashed lines) at the point of largest curvature (arrow). Note that the rod breaks at the point of maximal curvature.
Fast removing of the constrain from the extremity of the half-rod results in generation of the elastic wave:

FIG. 2. (a) Numerical solution of the Kirchhoff Eq. (1) with clamped-free boundary conditions, for a uniform initial curvature $\kappa_0$. The curvature at the free end $\kappa(0, t)$ relaxes to zero within the first few time steps (quick relaxation of the incompatible curvature near free end) while it is given in the intermediate regime (2) by the universal self-similar solution (4), shown in (b) as a function of $\xi = s/\sqrt{\gamma t}$. At later times, for $t \sim T$, reflections are generated from the clamped end $s = L$. 

Simul.mov
Chinese vapor cooked dumplings

\[ q(r, t) = -\kappa \frac{\partial T(r, t)}{\partial r} \]

\[ \frac{\partial T(r, t)}{\partial t} = \chi \Delta T(r, t) \]

\[ T(r = R, \forall t) = 100^0C \]

\[ T(r = 0, t = 0) = 20^0C \]

\[ \left( \frac{\partial T(r, t)}{\partial r} \right)_{r=R} \]
\[ q(R, t) = \left( \frac{\partial T(r, t)}{\partial r} \right)_{r=R} \]

\[ q(R, t) = r \delta m(t) = r \frac{\mu_{H_2O}}{N_A} \delta N(t) \]

\[ \delta N(R, t) = N_A \left[ \frac{\kappa}{r \cdot \mu_{H_2O}} \right] \left( \frac{\partial T(r, t)}{\partial r} \right)_{r=R} \]

\[ \delta N(R, t) + \delta n \]

\[ \delta N(R, t) - \delta n \]
5. Physics of a Good Coffee
Italian Moca
Filtration: the Darcy Law

\[ Q = \frac{\kappa \cdot S \cdot \rho \cdot \Delta P}{L \cdot \eta} \]

\[ \Delta P = Q \cdot \eta \cdot \frac{L}{(S \cdot \rho \cdot \kappa)} \]

\[ P = P_{\text{atm}} \]

\[ Q = \frac{m}{t} \sim 200 \text{ g/min} \]

\[ \eta(\,^\circ C) = 10^{-3} \text{ Pa s} \]

\[ S \sim 10 \text{ cm}^2 \]

\[ L \sim 1 \text{ cm} \]

\[ \kappa \sim 10^{-12} \]

\[ P(T) = P_{\text{atm}} + \Delta P \]

\[ \Delta P \sim 4 \cdot 10^4 \text{ Pa} \Rightarrow T^* \sim 110 \, ^\circ C \]

Table 12.1: Temperature dependence of pressure of saturated water vapor.

<table>
<thead>
<tr>
<th>Temperature, °C</th>
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<th>Temperature, °C</th>
<th>Temperature, °C</th>
<th>Temperature, °C</th>
<th>Temperature, °C</th>
<th>Temperature, °C</th>
<th>Temperature, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, kPa</td>
<td>96.18</td>
<td>99.1</td>
<td>99.6</td>
<td>99.9</td>
<td>100</td>
<td>101</td>
<td>101.3</td>
<td>105</td>
</tr>
<tr>
<td>88.26</td>
<td>98.07</td>
<td>100</td>
<td>101</td>
<td>101.3</td>
<td>105</td>
<td>147</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The limitation of the Darcy Law at small pressures: opening of the capillaries

Filtrazione

\[ \Delta P_{\text{min}} \approx \frac{2\sigma}{r} \]

\[ r \approx 0.1 \text{mm} \]

\[ \sigma = 0.072 \text{ N/m} \]

\[ \Delta P_{\text{min}} = 0.05 \text{ atm} \]
Espresso machine

\[ Q = \frac{\kappa \cdot S \cdot \rho \cdot \Delta P}{L \cdot \eta} \]
Ancestors of pizza in ancient times

Pharaonic birthdays in Ancient Egypt

Flat cakes with vegetables used to eat in the Near East and pharaonic Egypt
Ancestors of pizza in ancient Rome

In Pseudo Virgilio, poem “Il Moretum”, cooking of some kind of a flat cake dressed by olive oil, salt, garlic and spice herbs was described.
Mozzarella as the result of Longobardi invasion
Arrival of tomato’s from America to Naples due Spanish and Bourbon rule
Birth of the modern pizza in Naples at the end of XVIII
Royal visit of Naples in 1889
Beginning of XX: pizza emigration
Celebration of 100 years of Margherita in 1989 Naples
The facts to think

Pizza Napoletana: 460-490 °C .
Pizza Romana
325- 330 °C → 2 min

In rush time 330 °C → 390 °C .
40 -50 → 80-100 customers
Homogeneous temperature distribution

97 F = 36.1°C
Temperature profile in homogeneous media

The same but with a cooler steel “head”
Temperature profile in non-homogeneous media
Heat flow: a little bit of school physics

Specific heat capacity:

\[ c = \frac{\Delta Q}{M \Delta T} \]

M is the body mass, \( \Delta Q \) is the Heat transferred to it, \( \Delta T \) is the change of its temperature.

Heat flow:

\[ q = -\frac{\Delta Q}{S \Delta t} \]

\[ q = \frac{c M \Delta T}{S \Delta t} = c \rho \frac{(\Delta x)^2}{\Delta t} \left( \frac{\Delta T}{\Delta x} \right) = -\kappa \frac{dT}{dx} \]

\( \frac{\Delta T}{\Delta x} \) - “speed” of temperature change in the space

\( \kappa = c \rho \frac{(\Delta x)^2}{\Delta t} \)

Heat flow in a cylinder from hot \((T_0+DT)\) to cold \((T_0)\). Notice, the temperature decreases from left to right!
Propagation of the heat front

\[
\frac{\Delta Q}{S \cdot t} = \frac{c \cdot \rho \cdot L(t) \cdot \Delta T}{t} = \kappa \left( \frac{\Delta T}{L(t)} \right)
\]

Heat length

\[L(t) \sim \sqrt{\frac{\kappa t}{c \rho}} = \sqrt{\chi t},\]

\[\chi = \frac{\kappa}{c \cdot \rho}\] -- is the coefficient of the temperature conductivity

Exact answer is \[L(t) = \sqrt{\pi \chi t}.\]
Temperature at interface between two semispaces

\[ T_1(t) = \sqrt{\chi_1} \cdot t \]

\[ L_2(t) = \sqrt{\chi_2} \cdot t \]

\[ q = \kappa_1 \frac{T_1 - T_0}{\sqrt{\pi \chi_1 t}} = \kappa_2 \frac{T_0 - T_2}{\sqrt{\pi \chi_2 t}}. \]
# Interface temperature

\[ T_0 = \frac{T_1 + \nu T_2}{1 + \nu} \]

\[ \nu = \frac{\kappa_2}{\kappa_1} \sqrt{\frac{\chi_1}{\chi_2}} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Heat capacity (c) [J/(kg×K)]</th>
<th>Thermal conductivity (\kappa) [W/(m×K)]</th>
<th>Mass density (\rho) [kg/m³]</th>
<th>Temperature conductivity (\chi) [m²/s]</th>
<th>(\nu_{21})</th>
</tr>
</thead>
<tbody>
<tr>
<td>dough³</td>
<td>2-2.5×10³</td>
<td>0.5</td>
<td>0.6-0.8×10³</td>
<td>0.7×10⁻⁷</td>
<td>1</td>
</tr>
<tr>
<td>food grade steel (X18H10T)</td>
<td>4.96×10²</td>
<td>18</td>
<td>7.9×10³</td>
<td>4.5×10⁻⁵</td>
<td>0.1</td>
</tr>
<tr>
<td>fire brick</td>
<td>8.8×10²</td>
<td>0.86</td>
<td>2.5×10³</td>
<td>4.0×10⁻⁷</td>
<td>0.7</td>
</tr>
<tr>
<td>water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Wood oven: Dough/firebrick: $v_{Df} = 0.65$

$$T_1 = 330 \, ^\circ C, \ T_2 = 20 \, ^\circ C$$

$$T_{0w}^w = \frac{330 \, ^\circ C + 0.65 \cdot 20 \, ^\circ C}{1.65} \approx 208 \, ^\circ C.$$  

Electric oven with steel bottom: Dough/steel: $v_{DS} = 0.1$

$$T_1 = 330 \, ^\circ C, \ T_2 = 20 \, ^\circ C, \ T_0 = 280 \, ^\circ C$$

$$T_{0e}^{eo} = \frac{330 \, ^\circ C + 0.1 \cdot 20 \, ^\circ C}{1.1} \approx 300 \, ^\circ C.$$  

What temperature in oven would be good for pizza?

$$T_1 = 230 \, ^\circ C, \ T_2 = 20 \, ^\circ C, \ T_0 = 208 \, ^\circ C$$
Three ways of heat transfer
Thermal radiation

Its intensity, i.e., the amount of radiation energy arriving each second to 1cm² of surface in the oven, is determined by the Stefan-Boltzmann law:

\[ I = \sigma T^4 \]

\[ \sigma = 5.67 \cdot 10^{-8} \text{W} / (K^4 \cdot m^2) \]

is the so-called Stefan-Boltzmann constant.
Radiation contribution for baking pizza

WOOD OVEN

\[ T_{1}^{wo} = 330^\circ C = 603 K, \]
\[ I^{wo} = \sigma (T_{1}^{wo})^4 = 5.67 \cdot 10^{-8} (603)^4 \]
\[ = 4.4 \cdot 10^4 W/m^2, \]

ELECTRIC OVEN

For the much less heated electric oven (t=230 \(^0\)C=503 K) the corresponding amount of energy, incident to 1cm\(^2\) of pizza surface, is more than twice less:

\[ I^{eo} = \sigma (T_{1}^{eo})^4 = 5.67 \cdot 10^{-8} (503)^4 = 2.1 \cdot \frac{10^4 W}{m^2}, \]
Here one should notice, that, in its turn, the pizza also irradiates out a “flow” of the intensity

\[ I_{pizza} = \sigma (T_{pizza})^4. \]

Since the major part of the baking time is required for the evaporation of water contained in the dough and toppings, we can assume

\[ T_{pizza} = 100^\circ C = 373 \, K, \]

which results in a radiation intensity of

\[ I_{pizza} = \sigma (T_b)^4 = 0.66 \times 10^4 \, W/m^2, \]

i.e., 15% of the obtained radiation, the pizza “returns” to the oven.
The amount of heat 1 cm\(^2\) of pizza receives per second through its bottom

\[
q(t) = \kappa \frac{T^o - T_0}{\sqrt{\pi \chi t}},
\]

\[
Q(\tau) = \int_0^\tau q(t) dt = 2\kappa (T^o - T_0) \sqrt{\frac{\tau}{\pi \chi}}.
\]

Total amount of heat 1 cm\(^2\) of pizza receives per second

\[
Q_{tot}(\tau) = \sigma \left[ (T_{1}^o)^4 - (T_b)^4 \right] \tau + 2\kappa (T^o - T_0) \sqrt{\frac{\tau}{\pi \chi}}.
\]

This value is used to heat 1 cm\(^2\) of pizza from 20 \(^0\)C to the boiling temperature of water 100 \(^0\)C

\[
Q_{heat} = c^{do} \rho^{do} d(T^b - T_0^{do}).
\]
During the process of baking the perfect pizza we apparently evaporate water from the dough, tomatoes, cheese, and other ingredients. We need to take the required energy for this into account as well. If one assumes that the mass fraction \( \alpha \) (which is water, for Roman pizza \( \alpha = 20\% \)) evaporates from the dough and all topping one gets:

\[
Q_{boil} = \alpha L \rho_{\text{water}} d.
\]

Collecting both these contributions in one, we can write

\[
Q_{\text{tot}} = Q_{\text{heat}} + Q_{boil} = c^{do} \rho^{do} d (T_b - T_0^{do}) + \alpha L \rho_{\text{water}} d.
\]

Equating Eqs. (11) and (12) one finds the final equation determining the "baking of pizza:

\[
\sigma \left[ (T_1^o)^4 - (T_b^o)^4 \right] \tau + 2 \kappa (T_0^o - T_0) \sqrt{\frac{\tau}{\pi \chi}} = c^{do} \rho^{do} d (T_b^b - T_0^{do}) + \alpha L \rho_{\text{water}} d.
\]

\[
\tau_{wo} \approx 115 \text{ s}.
\]

\[
\tau_{eo} \approx 240 \text{ s}.
\]
Смачного!