Dynamics of solitons in low-dimensional nanosystems under the influence of external oscillating magnetic field

K. Temchenko, L. Brizhik

NTUU “Kyiv Polytechnic Institute”, Bogolyubov Institute for Theoretical Physics

Introduction

In this work, the influence of oscillating magnetic field on the dynamics of Davydov’s soliton is investigated. Soliton is formed in a molecular chain due to electron-phonon interaction. It describes the self-trapped state of an excess electron in the deformation potential.

Here we will not study the effect of magnetic field on electron spin, because we consider low intensity fields, and because solitons exist at speeds less than the velocity of sound in the chain. Under these conditions, the spin-orbit interaction is insignificant.

Soliton motion in the longitudinal magnetic field

In this case we can choose the wave function in the form:

\[ \psi(x, t) = \phi(x) e^{-i \omega t} \]

Equations of motion:

\[ \frac{\partial^2 \phi}{\partial t^2} + \left( \frac{\omega^2}{c^2} \right) \phi = \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} = 0 \]

Here \( \phi(x) \) is the wave function of a free soliton, and for \( \psi(y, z, t) \) we have:

\[ \psi(y, z, t) = \frac{1}{\sqrt{4\pi i}} \exp\left[ -\frac{1}{2} \left( y^2 + z^2 \right) \right] H(y) \exp\left[ \frac{i}{2} y^2 \right] \]

Thus, the electron cyclotron frequency in the transverse direction depends on the intensity and frequency of the magnetic field.

Soliton motion in the transverse magnetic field

Soliton dynamics in this case can be described by the dimensionless nonlinear Schrödinger equation with the right hand side that is defined by the field:

\[ i \frac{\partial \phi}{\partial t} + \frac{\omega^2}{c^2} \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} + \frac{1}{2} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} = 0 \]

The extra term can be considered as a small perturbation as compared with the terms of the nonlinear Schrödinger equation. Therefore, soliton dynamics can be investigated by perturbation theory for nonlinear equations developed within the method of the inverse scattering problem.

The solution of the equations of motion is:

\[ \psi(x, t) = 2\nu \text{sech} \left[ 2\nu (x - \xi) \right] \exp \left[ 2i\mu \left( x - \xi \right) + i\nu \right] \]

Here, due to the perturbation, soliton parameters depend on time:

\[ \nu = C_x, \quad \xi = 2\nu t + C_x, \quad \mu = -i \frac{\xi}{2\Omega} (\Omega + \sin \Omega \cos \Omega) + \frac{\xi}{\Omega} \sin \Omega + \frac{\nu}{\Omega} + C_x, \quad \eta = 2i \left( \nu^2 + \mu^2 \right) + i \left[ \frac{\xi^2}{4\Omega^2} + \frac{\xi}{\Omega} \sin \Omega + \frac{\nu}{\Omega} + C_x \right] \frac{\Omega^2}{2} \sin \Omega \cos \Omega + \frac{2\nu}{\Omega} \sin \Omega - 2\nu^2 + C_x \]

Free molecular soliton

Consider a molecular chain with an excess electron, taking into account the electron-phonon interaction.

Soliton dynamics in this case can be described by the dimensionless nonlinear Schrödinger equation with the right hand side that is defined by the field:

\[ \frac{\partial \phi}{\partial t} + \frac{\omega^2}{c^2} \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} + \frac{1}{2} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} = 0 \]

The extra term can be considered as a small perturbation as compared with the terms of the nonlinear Schrödinger equation. Therefore, soliton dynamics can be investigated by perturbation theory for nonlinear equations developed within the method of the inverse scattering problem.

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Conclusions

- Periodic magnetic field changes the dynamics of solitons. Effect of the magnetic field depends on its orientation with respect to the chain.
- In the longitudinal magnetic field soliton dynamics consists of electron motion along the chain in the state of a free soliton and the motion in the transverse direction to the chain, which is described by functions of harmonic oscillator.
- In a transverse magnetic field parameters of molecular soliton, including its speed, width and energy, are oscillating functions of time with the external field frequencies and higher harmonics.
- Account of energy dissipation results in the decrease of the soliton velocity (deceleration) due to the magnetic field with established speed balance.
- Such a complex impact of magnetic field on the dynamics of solitons causes change electron transport processes in low-dimensional molecular systems and macromolecules.

Dynamics in the magnetic field with account of energy dissipation

Molecular chains are always in some environment that leads to energy dissipation and deceleration of solitons. Here we consider transverse oscillating magnetic field.

With account of energy dissipation, the equation for the wavefunction is:

\[ \frac{\partial \psi}{\partial t} + \frac{\omega^2}{c^2} \psi + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \frac{1}{2} \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} = 0 \]

The solution can be found using the perturbation theory, based on the method of the inverse scattering problem:

\[ D(x, t) = D(x, t) + \delta D(x, t), \quad D(x, t) = 2\nu \text{sech} \left[ 2\nu (x - \xi) \right] \exp \left[ 2i\mu (x - \xi) + i\nu \right] \]

Soliton parameters are defined by equations of the perturbation theory.