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Effect of interactions on quantum limited detectors

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PLAN

1. Introduction: measurement-caused qubit decoherence and acquisition information rate for quantum detector.

2. What was already known before about charge-qubit interacting with QPC and what new we have done with it.

3. The role of electron-electron interactions: model Hamiltonian.

4. Qubit’s decoherence rate: “Orthogonality catastrophe” and “bias voltage” contributions.


6. Full Counting Statistics (FCS) of a Luttinger liquid QPC.

7. Acquisition information rate for LL QPC as a quantum detector.

8. Luttinger liquid QPC as a quantum-limited detector of a qubit’s state: “sensibility” (or detector “efficiency”) rate.

9. The crucial role of the “orthogonality catastrophe” effect for QPC’s detector properties.

10. Conclusions.
Measurement-caused decoherence and detection of a qubit’s state: different realizations

Interaction Hamiltonian

Acquisition information

Qubit decoherence

Qubit

Signal

Back action

Quantum detector

Concept of charge qubit: decoherence of the electron state in Josephson junction due to interaction with FL QPC (Aleiner, Wingreen, Meir, PRL, 1997)

Well-known examples

- Known for the case of “free” fermions only (!) - QPCs with Fermi liquid leads (!)

- However “in reality” electron-electron interaction in QPC matters (!)

General concept of a QPC as a quantum limited detector (Averin, Sukhorukov PRL, 2005)
QUBIT DENSITY MATRIX: ITS TIME EVOLUTION AND DECOHERENCE

\[ \tilde{\rho}_{mn}(t) = \rho_{mn}(0) \langle \chi(t) | \bar{U}_n(t) U_m(t) | \chi(t) \rangle \]

\[ U_n(t) = T_t \exp \left\{ -i \int_0^t d\tau \tilde{H}_{int}^{(n)}(\tau) \right\} \]

\[ \bar{U}_n(t) = T_t \exp \left\{ i \int_0^t d\tau \tilde{H}_{int}^{(n)}(\tau) \right\} \]

\[ \rho(0) = |\phi_0\rangle \langle \phi_0| \]

\[ H_{QD} |n\rangle = \varepsilon_n |n\rangle \]

NONINTERACTING ELECTRONS IN THE QPC: Aleiner, Wingreen Meir result (PRL, 1997) for qubit being coupled to Fermi Liquid QPC (in the "weak backscattering" limit)

\[ A_+(t) = \left( \frac{i \pi T}{\xi_0 \sinh \pi T t} \right)^{\alpha + \gamma} e^{-\Gamma_d t + \gamma h(t, T, eV)} \]

\[ \alpha = 4 \left( \frac{\delta \phi}{\pi} \right)^2 \]

\[ \Gamma_d = \pi \gamma |eV| \]

\[ \gamma = 4 \lambda_x^2 \cos^4 \delta \phi \]
ACQUISITION INFORMATION RATE FOR THE OVERLAP OF THE TWO MARKOVIAN DISTRIBUTIONS ON THE LARGE TIMESCALE: DEFINITION

\[ \tilde{M}_{mn}(N) = \sum_{N} \sqrt{P_m(N)P_n(N)} \quad m, n = 1, 2 \]

\[ \Lambda_g^{-1} \ll t \ll \gamma^{-1} \to \infty \]

\[ F_M(\xi, t) \propto t \]

\[ t = \hbar N/\Lambda_g \to \infty \]

\[ M_{mn}(t)_{t \to \infty} = M_{nm}(t)_{t \to \infty} = \exp\{-W_I \cdot t\} \]

\[ W_I = W_I(\xi_c) = \min_{\xi = \xi_c} \{W_I(\xi)\} \]

\[ W_I(\xi) = \frac{1}{2} \{W_1(\xi) + W_2(-\xi)\} \]

Most general Averin, Sukhorukov result (PRL, 2005) for quantum limited FL QPC detector in a weak backscattering limit — e.g. for Mach-Zender interferometer measurements

- For any quantum limited detector(!)

\[ W_{jk} \leq \Gamma_{jk} \]

\[ W_{jk} = \Gamma_{jk} = -\frac{eV}{2\pi\hbar} \ln[(D_jD_k)^{1/2} + (R_jR_k)^{1/2}] \]

At T=0 in “quantum limit”
MODEL HAMILTONIAN

\[
H_\Sigma = H_{LL} + H_{QD} + H_{int}
\]

\[
H_{LL} = \frac{1}{2\pi} \sum_{j=L,R} v_g \int_{-\infty}^{0} \left\{ g \left( \partial_x \varphi_j \right)^2 + \frac{1}{g} \left( \partial_x \theta_j \right)^2 \right\} dx
\]

\[
H_{QD} = \sum_{n=1,2} \varepsilon_n c_n^\dagger c_n + \gamma \left( c_1^\dagger c_2 + c_2^\dagger c_1 \right)
\]

\[
H_{int} = \sum_{n=1,2} \left[ \lambda_n \partial_x \theta_+ + \tilde{\lambda}_n \cos (\varphi_+ + eVt) \right] |_{x=0} c_n^\dagger c_n
\]

Hamiltonian of two semi-infinite Luttinger liquids

Charge-qubit part

Tunnel Hamiltonian in the “weak link” approximation (Kane, Fisher, PRB, 1992)

\[
\theta_\pm = [\theta_L \pm \theta_R]
\]

\[
\varphi_\pm = [\varphi_L \pm \varphi_R]
\]

Non-local bosonic phase fields (the important ingredient)

\[
[\theta_\alpha(x), \varphi_\alpha'(x')] = 2i\pi \text{sgn}(x - x') \delta_{\alpha,\alpha'}
\]

\[
[\theta_\alpha(x), \partial_x \varphi_\alpha'(x')] = 2i\pi \delta(x - x') \delta_{\alpha,\alpha'}
\]

\[
\alpha = \pm
\]
REDUCED DENSITY MATRIX PROPAGATOR: ITS FACTORIZATION ON "ORTHOGONALITY CATASTROPHE" AND "TUNNEL" CONTRIBUTIONS

\[ [\varphi_+(t), \varphi_-(t')] = 0 \]

\[ \tilde{\rho}_{mn}(t) = \rho_{mn}(0) Z_{(mn)}(t) \tilde{Z}_{(mn)}(t) \]

\[ Z_{(mn)}(t) = \left\langle \exp \left\{ i \frac{g(\lambda_n - \lambda_m)}{v_g} [\varphi_+(t) - \varphi_+(0)] \right\} \right\rangle \]

\[ \tilde{Z}_{(mn)}(t) = \langle \overline{T}_t \exp \left\{ i\tilde{\lambda}_n \int_0^t d\tau \cos [\varphi_-(\tau) + eV\tau] \right\} \times \right. \]
\[ \left. T_t \exp \left\{ -i\tilde{\lambda}_m \int_0^t d\tau \cos [\varphi_-(\tau) + eV\tau] \right\} \right\rangle \]

- "Orthogonality catastrophe" term
- "Tunnel" contribution
ORTHOGONALITY CATASTROPHE CONTRIBUTION TO DECOHERENCE

\[ \langle \varphi_+(t) \rangle = 0 \]

\[ Z_{(mn)}(t) = \exp \left\{ -\frac{1}{2} \left( \frac{g (\lambda_n - \lambda_m)}{v_g} \right)^2 \langle \Delta \varphi_+ (t)^2 \rangle \right\} \]

\[ \Delta \varphi_+ (t) = \varphi_+(t) - \varphi_+(0) \]

\[ Z_{(12)}(t) = Z_{(21)}(t) = \left[ \frac{\pi T/\Lambda_g}{\sinh(\pi T \cdot t)} \right]^{2g(\Delta \lambda/\Lambda_g)^2} \]

\[ \Delta \lambda = \lambda_1 - \lambda_2 \quad \Lambda_g = \Lambda_0/g \quad \Lambda_0 \simeq \varepsilon_F \]
“TUNNEL” DECOHERENCE RATE IN LUTTINGER LIQUID CASE: GENERAL (EXACT) FORMULA

\[ \Lambda_g^{-1} \ll t \lesssim \gamma^{-1} \rightarrow \infty \]

\[ \tilde{Z}_{12(21)}(t)_{|t|\gg 1/\Lambda_g} = \exp(-\Gamma_t \cdot t) \]

\[ \Gamma_t = \frac{(\tilde{\lambda}_1 - \tilde{\lambda}_2)^2}{\Lambda_g} F_g(eV, T) \cosh(eV/2T) \]

\[ F_g(eV, T) = \left| \frac{\Gamma (1/g + i [eV/2\pi T])]}{\Gamma (2/g)} \right|^2 \left[ \frac{2\pi T}{\Lambda_g} \right]^{(2/g-1)} \]
Plasmonic mechanism of decoherence in Luttinger liquid QPC: strong suppression of a qubit decoherence rate: qualitative picture

“Non-interacting” case: $g=1$

The “overlap” integral between wave functions of two “plasmonic cloud-dressed” states of a qubit.

$\tau_{0q} \propto \exp(n \cdot N_q)$

“Interacting” case: $g=(1/n)<1$, $n=3$

A “plasmonic overlap”, representing a product of “n” overlaps between partially charged ($q=1/n$) plasmonic “clouds”, is a Luttinger liquid generalization of the overlap between two “plasmonic cloud dressed” states of the qubit.
FCS-CGF AS AVERAGE OF THE KELDYSH-CONTOUR-ORDERED EXPONENTIAL WITH QUANTUM POTENTIAL: LUTTINGER LIQUID QPC

\[
\tilde{\chi}_{1(2)}(\xi, t) = \left\langle \mathcal{T}_K \exp(-i\tilde{\lambda}_{1(2)} \int_{\mathcal{C}_K} A_{\xi(\tau)}(\tau) d\tau) \right\rangle
\]

\[
A_{\xi(\tau)}(\tau) = \cos \left[ \varphi_-(\tau) + f_{\xi(\tau)}(\tau) \right]
\]

\[
\xi(\tau) = \pm \xi \quad \xi(\tau) \in (-\pi; \pi)
\]

\[
A_{\xi(\tau)}(\tau) = \begin{cases} 
A_{+\xi}(\tau) = \cos \left[ \varphi_-(\tau) + f_{+\xi}(\tau) \right], \text{Im}\{\tau\} > 0 \\
A_{-\xi}(\tau) = \cos \left[ \varphi_-(\tau) + f_{-\xi}(\tau) \right], \text{Im}\{\tau\} < 0
\end{cases}
\]

\[
f_{\pm\xi}(\tau) = [eV\tau \pm \xi]
\]
$W_{1(2)}(\xi) = \{ \Gamma_{g,+} \left[ e^{2i\xi} - 1 \right] + \Gamma_{g,-} \left[ e^{-2i\xi} - 1 \right] \}$

$\Gamma_{g,\pm} = \Gamma_{g,\pm}(eV, T) = \left( \frac{\tilde{\lambda}_{1(2)}^2}{4\Lambda_g} \right) F_g(eV, T)e^{\pm eV/2T}$

$\frac{\Gamma_{g,\pm}(eV, T)}{\Gamma_{g,\mp}(eV, T)} = e^{\pm eV/T}$

$\langle \langle N^k(t) \rangle \rangle_{1(2)} = \begin{cases} -(2e)^k (\Gamma_{g,+} + \Gamma_{g,-}) \cdot t & , k = even \\ -(2e)^k (\Gamma_{g,+} - \Gamma_{g,-}) \cdot t & , k = odd \end{cases}$

$\bar{I}_{g,1(2)}(eV, T) = -e \left( \frac{\tilde{\lambda}_{1(2)}^2}{\Lambda_g} \right) F_g(eV, T) \sinh(eV/2T)$

$\bar{S}_{g,1(2)}(eV, T) = -2e^2 \left( \frac{\tilde{\lambda}_{1(2)}^2}{\Lambda_g} \right) F_g(eV, T) \cosh(eV/2T)$
(EXACT) ACQUISITION INFORMATION RATE FOR THE LUTTINGER LIQUID
QPC: EXPLICIT EXPRESSION

\[ W_I = F_g(eV, T) \cosh(eV/2T) \times \left\{ \lambda_+ - \sqrt{\lambda_+^2 - (\tanh[eV/2T] \lambda_-)^2} \right\} \]

\[ F_g(eV, T) = \frac{|\Gamma (1/g + i [eV/2\pi T])|^2}{\Gamma (2/g)} \left[ \frac{2\pi T}{\Lambda_g} \right]^{(2/g-1)} \]

\[ \lambda_\pm = \left( \frac{\tilde{\lambda}_1^2 \pm \tilde{\lambda}_2^2}{\Lambda_g} \right) \]
QUANTUM DETECTOR “EFFICIENCY” (OR “SENSIBILITY”) RATE FOR THE LUTTINGER LIQUID QPC AS SUCH DETECTOR: GENERAL (AND EXACT) FORMULA

\[ Q \equiv \frac{W}{\Gamma_{\text{tot}}} = \frac{W}{(\Gamma + \tilde{\Gamma})} \leq 1 \]

\[ \eta = \frac{(\tilde{\lambda}_2 - \tilde{\lambda}_1)}{\left(\tilde{\lambda}_1 + \tilde{\lambda}_2\right)} \]

\[ Q = \frac{1 + \eta^2}{2\eta^2} \left[ 1 - \sqrt{1 - \left(\frac{2\eta}{1 + \eta^2} \tanh(eV/2T)\right)^2} \right] \frac{1}{\left(1 + \Gamma/\tilde{\Gamma}\right)} \]

-For example, if: \( T \gg eV \)

and: \( t \gg 1/\pi T \)

Then:

\[ \frac{\Gamma}{\tilde{\Gamma}} \approx \left(\frac{\lambda_1 - \lambda_2}{\tilde{\lambda}_1 - \tilde{\lambda}_2}\right)^2 \frac{2}{\sqrt{\pi}} \left(\pi \frac{T}{\Lambda_g}\right)^{2-\frac{2}{g}} \sqrt{g} \]
FIG. 3. (Color Online) Log-plot of the ratio between local density- and tunneling-induced decoherence as a function of (a) bias voltage and (b) temperature, cf. Eqs. (11), (14). For increasing interaction strength $g = 1; 0.9; 0.6; 0.5; 0.3$ from light to dark red and from continuous to coarsely dashed. $\frac{\Gamma}{\hat{\Gamma}} = 0.01$ in (a) and $\frac{\tilde{g} \cdot \nu}{\lambda_{g}} = 0.01$ in (b), $\lambda_{2} - \lambda_{1} = \lambda_{2} - \lambda_{1} = \Delta \lambda$.

FIG. 5. (Color Online) Total detector efficiency $Q$ for the interacting case, as function of the applied bias (a) and temperature (b). Different curves from light to dark red and from continuous to coarsely dashed are for increasing interaction strengths, $g = 1; 0.9; 0.6; 0.5; 0.3$. The figures show strong dependence of $Q$ on $g$ once orthogonality effects are considered. The inset in (b) shows a zoom in of the regime of cross over between monotonous and non-monotonous temperature dependence for $g \sim 0.5$ ($g = 0.55; 0.525; 0.5; 0.475; 0.45$ from light to dark blue and from continuous to coarsely dashed). We used $\eta = 0.5$, and $\frac{T}{\Lambda} = 0.01$ in (a), $\frac{\tilde{g} \cdot \nu}{\Lambda} = 0.01$ in (b).
Main Results

1. Exact decoherence rate and “orthogonality catastrophe” coefficient for charge qubit being coupled to Luttinger liquid QPC with arbitrary electron-electron interaction in a weak tunneling limit.

2. Exact cumulant generating function (CGF) and full counting statistics for the Luttinger liquid QPC in a weak tunneling limit.

3. Novel effect: “Plasmonic” suppression of a qubit decoherence rate due to “coherent plasmonic-clouds” formation in the Luttinger liquid QPC.

4. General formulas for quantum detector “sensibility” (or “efficiency”) ratio in the case of Luttinger liquid QPC.

5. The crucial role of the “orthogonality catastrophe” effect for the detector properties of Luttinger liquid quantum point contacts.